

## INTRODUCTION

A mathematical model to estimate the effect to returning the rejection of a desalination process by reverse osmosis in an aquifer was developed. The system is based on an aquifer, membrane and storage tank. Two possible situations are assumed and compared: a long term drought of an aquifer with no recharge and the case when the aquifer has a recharge, in both cases (which are extreme situations) the concentration of solute in an aquifer is analyzed. An evaluation of the stability using local analysis of the stationary states is presented. The results obtained using in situ aquifer data of the Chaco Boreal show that the aquifer stabilizes to a stable hyperbolic sink. The results encourage the use of this methodology to analyze multistage systems, as well as, more complex model for the desalination aquifer system.

## ENVIRONMENTAL CONDITIONS

Environmental conditions are referred to the extremes that can be found in the environment characteristic of the Paraguayan Chaco. In rainy season a continuous recharge of the aquifer it is performed, during the drought season the aquifer is considered nule.



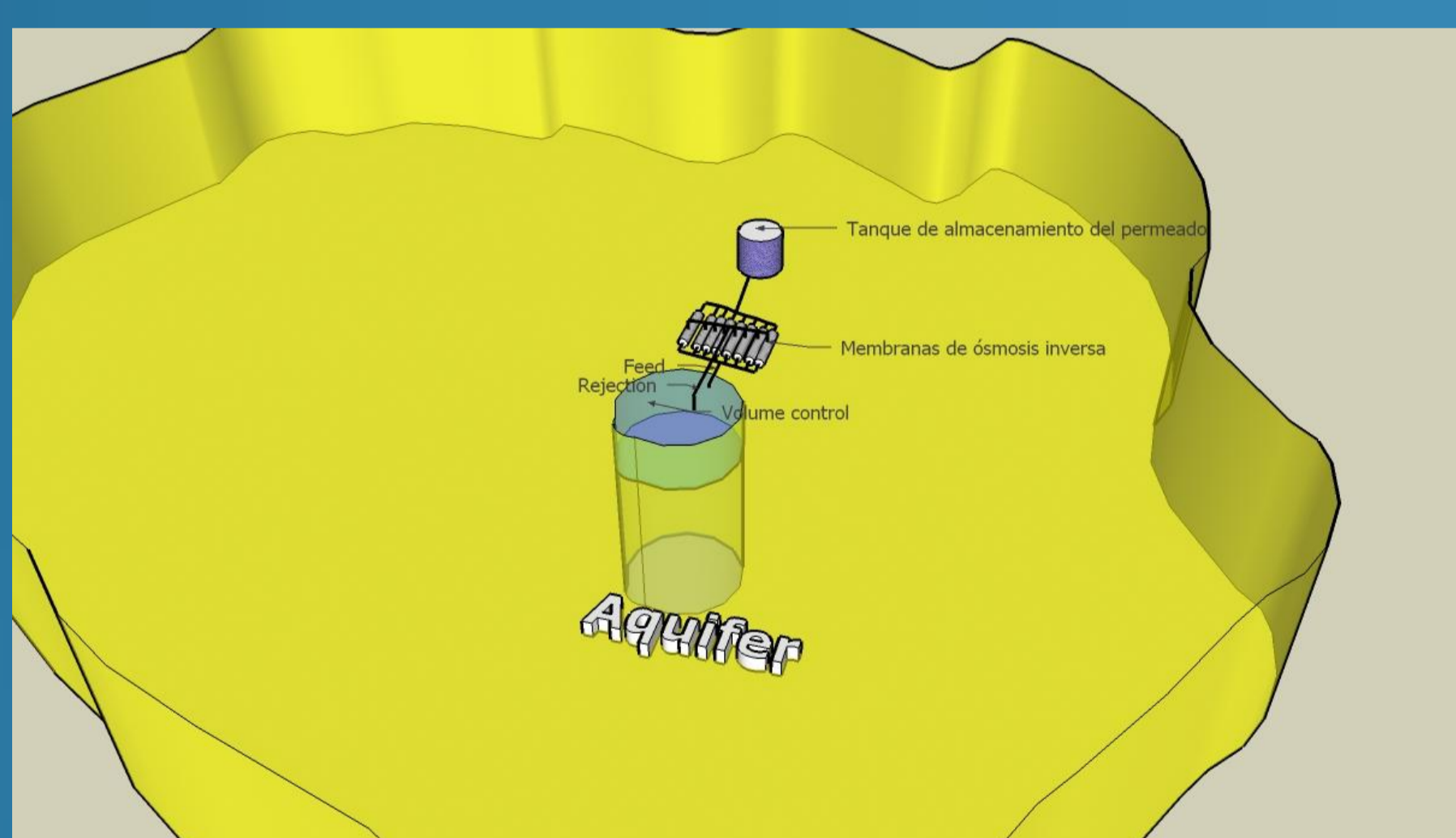
Rainy season



Dry season

## CONDITIONS OF THE MODEL

For the aquifer, the conditions are divided into :



Considered volume control

### Aquifer own

- The aquifer acts as a perfectly reactor (the concentration of solute in any part of the aquifer is the same for the volume control).
- The solution is considered as single component of dissolved sodium chloride (the major component in the waters of Chaco) and the entire system is at constant temperature.
- The recharge of the aquifer is represented by a constant flow with a constant concentration of salts (dissolved by water permeating through the soil) and this charge is entirely due to rains.

### Characteristic of reverse osmosis

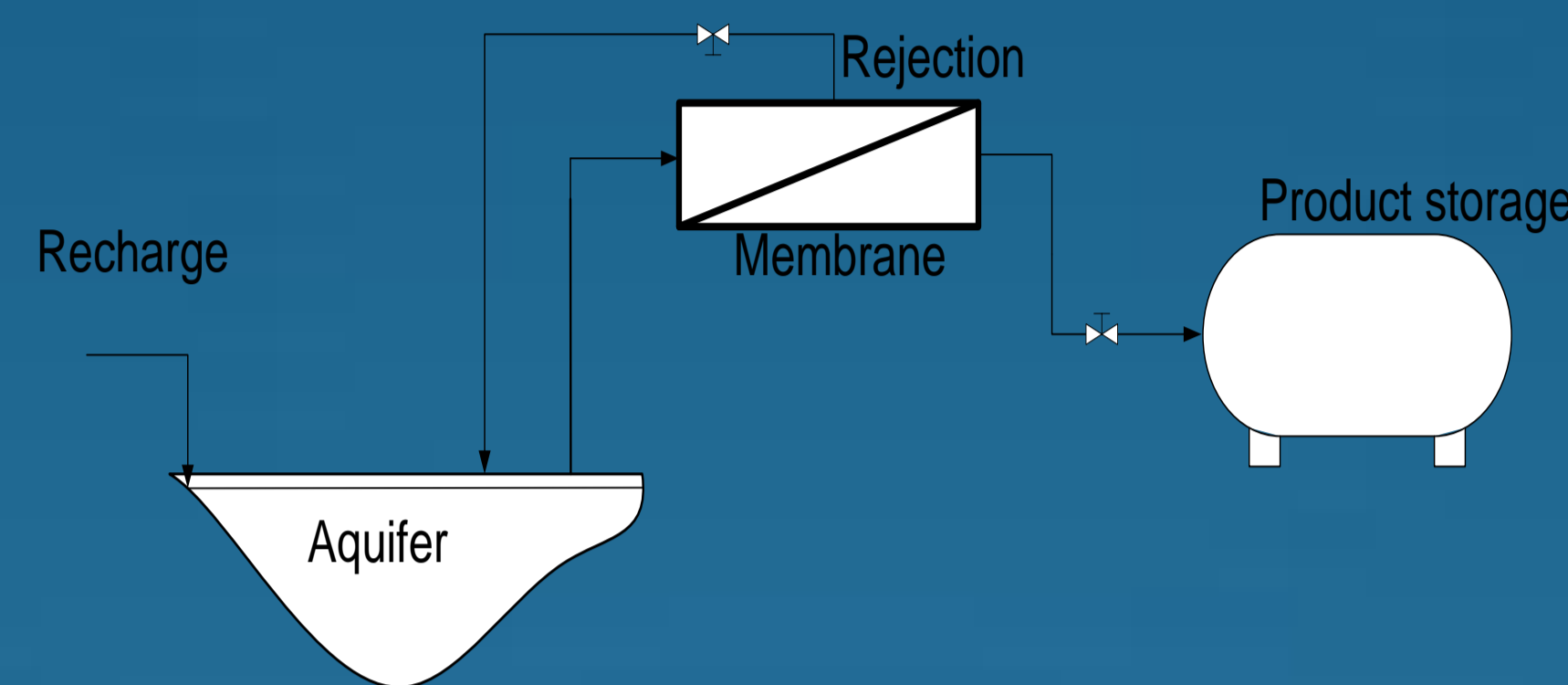
The mass transfer model is based on Darcy's law, which considers an ideal mass transfer through the pores of the membrane as a dominant phenomenon [2].

## BUILDING THE MODELS

We set up situations for which you will obtain the mathematical models:

### One step model

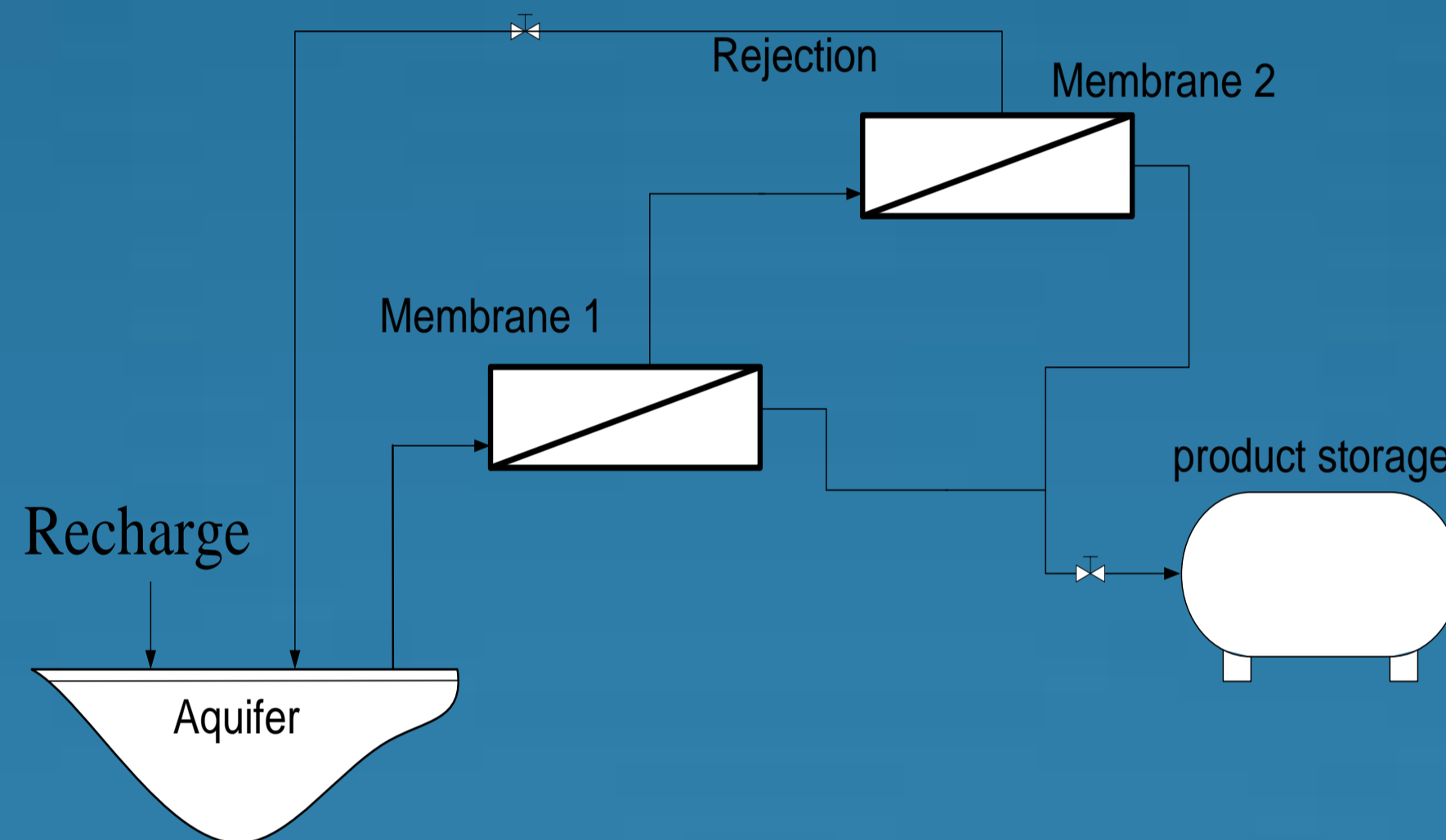
We consider the separation by reverse osmosis stage for a situation of extreme drought (Dry season) and a continuous rain (Rainy season).



Desalination system from one stage to aquifer recharge

### Two step model

For the environmental conditions considered, we obtain models for two stages of separation.



System with two-stage desalination of aquifer recharge

## EQUATIONS OF THE MODEL

For models we considerate two types of systems:

### Closed systems (Dry season)

As the temperature is considered constant for this system there is no exchange of matter or energy with the external environment, so the total amount of matter as the solvent (water) and solute (salt) remains constant. The mass balance of the system is presented below;

$$v_{fo}c_{f0} = v_f c_f + v_a c_a,$$

where the product  $v_i c_i$  ( $i$  represent the initial feed  $f_0$ ,  $f$  the final feed and  $a$  in the storage tank).

### Open system (Rainy season)

For this system there is an external input of material (aquifer recharge), which increases the total amount of matter over time. The material balance in the system is ;

$$q_n t + v_{fo} c_{f0} = v_f c_f + v_a c_a,$$

where the product  $q_n t$  is the increase in mass whit time.

### Mass transfer across the membrane

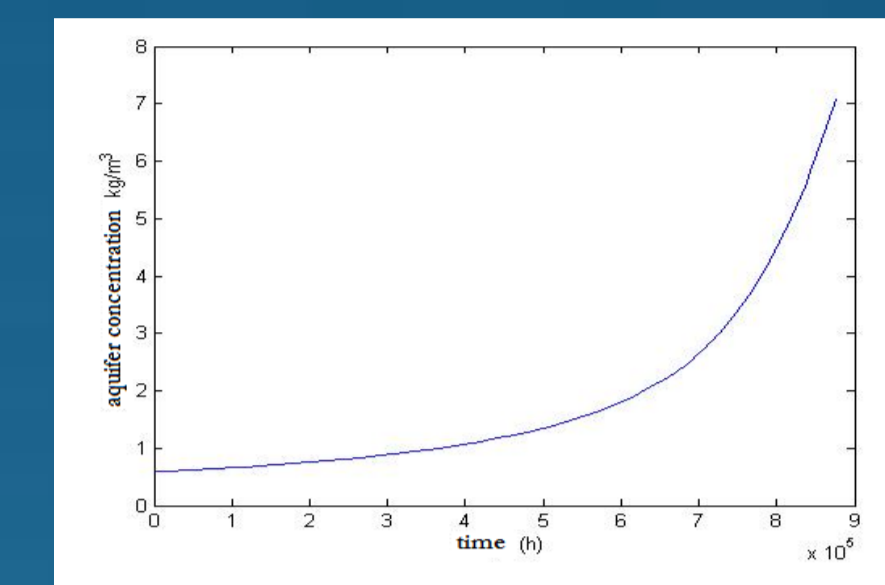
The mass transfer is represented by Darcy's Law [2][3];

$$\begin{cases} J_w = k_w (|P_f - P_p| - |\pi_f - \pi_p|), \\ J_s = k_s (c_f - c_p), \end{cases}$$

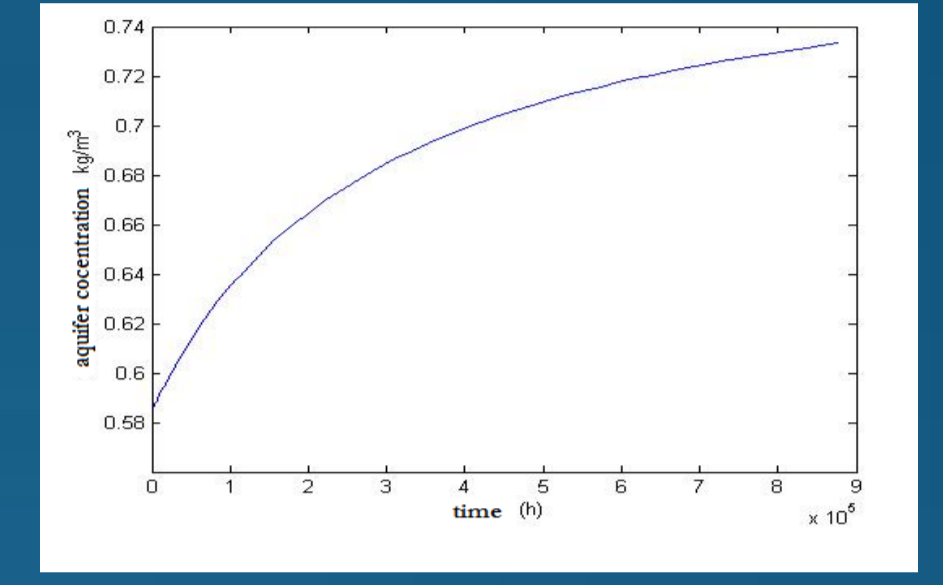
## NUMERICAL RESULTS

We simulate the interaction between the groundwater and desalination system for dry seasons and wet seasons for a period of 100 years and data extracted from the literature [3][4].

### One step

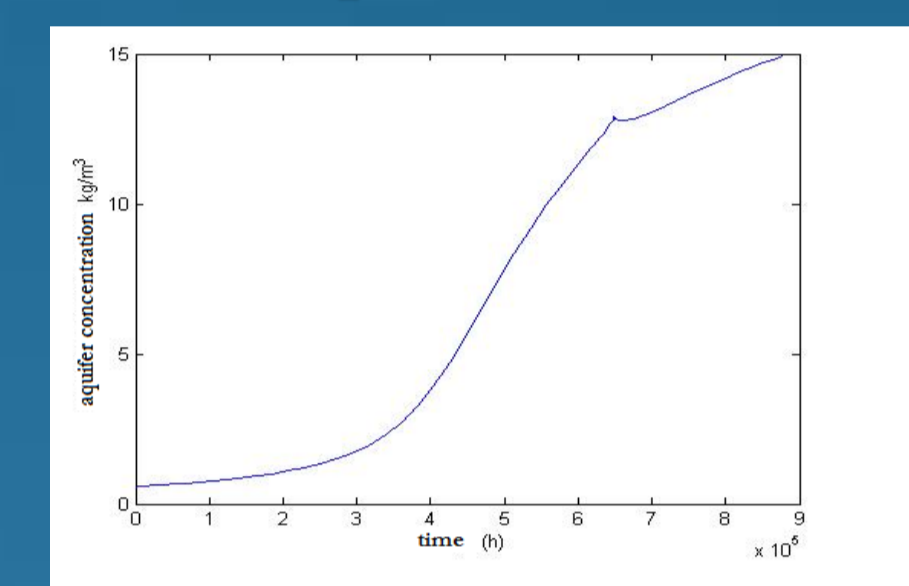


One stage with non-recharge: aquifer concentration respect to the time.

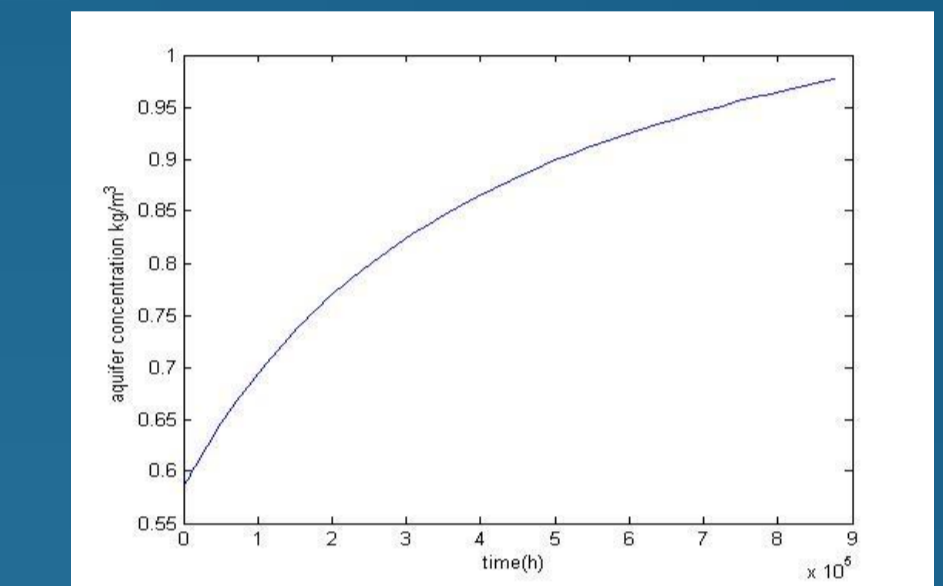


One stage with recharge: aquifer concentration respect to the time.

### Two Step



One stage with non-recharge: Two stage without recharge: aquifer concentration respect to the time.



Two stage AMT with recharge: aquifer concentration respect to the time.

## STABILITY

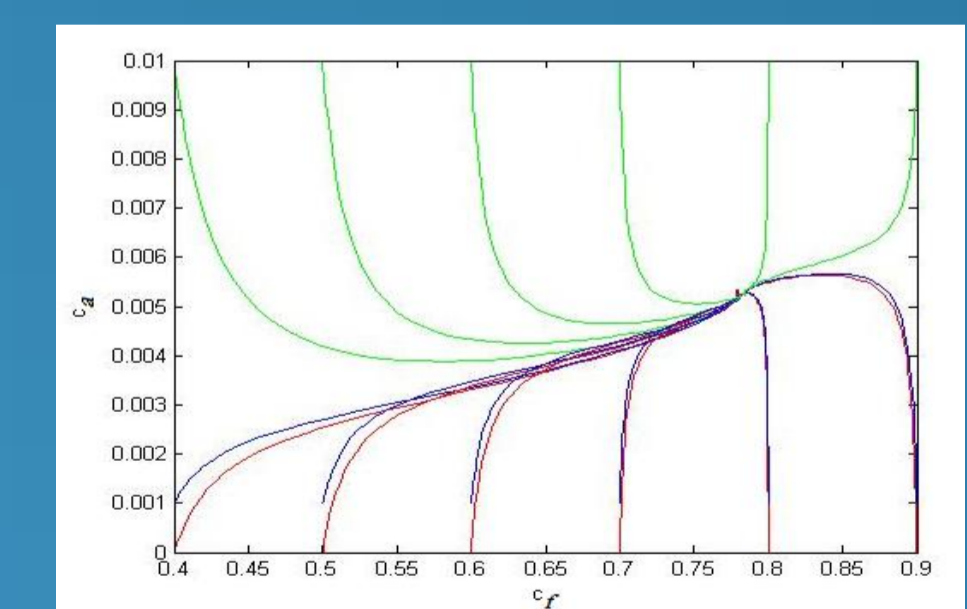
We analyzes the stability of the aquifer with recharge. The critical state of equations can be obtained setting the derivatives and equals to zero, and solving the resulting nonlinear system of equations:

$$\begin{cases} G_1(c_f, c_a, t) = 0, \\ G_2(c_f, c_a, t) = 0. \end{cases}$$

The nature of the critical state is analyzed using the eigenvalues  $\lambda$  and eigenvectors  $w$  of the Jacobian matrix

$$A = \begin{bmatrix} \partial G_1 / \partial c_f & 0 \\ \partial G_2 / \partial c_f & \partial G_2 / \partial c_a \end{bmatrix}$$

Is observed that the critical state is a stable sink attractor.



Phase portrait and state

## CONCLUSIONS

In this context, during the dry season (rainy season) it is preferable to use the model of the AMT system without the aquifer recharge (with recharge).

When the aquifer has a recharge, the system can reach an equilibrium state where the concentration in the aquifer depends on the aquifer recharge and the multistage desalination system. The qualitative analysis of this paper gives an idea about the operation point of the system, consequently, the energy and configuration necessary to design a system for obtaining a volume of fresh water with a specific salt concentration.

## REFERENCES

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